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# Parametric Study of the Radiated Field from Base Stations Using the Integral Equation Approach

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## Introduction

- Analysis of base station antenna systems can be carried out using a number of approaches with different degree of accuracy depending on the purpose of a particular study being undertaken.
- Provided a current distribution is determined along the wire one may evaluate the radiated field for the dosimetry purposes, i.e. for assessing the human exposure to a particular radiation source.

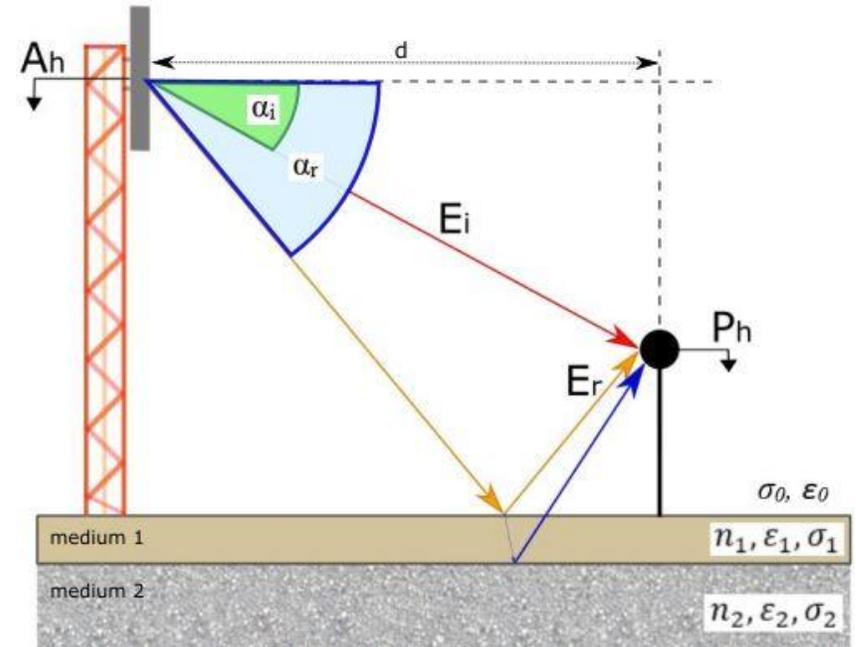


Figure 1. Radiation from base station antenna system

## Introduction

- The simplest and most commonly used approach is the ray tracing approximation being valid only in the far-field region. However, in some scenarios near-field zone is of interest and integral equation approach should be used.
- Rigorous approaches are based on the integro-differential equations of Pocklington and Hallen type in which currents along the wires are unknown. In order to obtain an unknown current distribution a governing equation needs to be numerically solved.

## Geometry of the problem

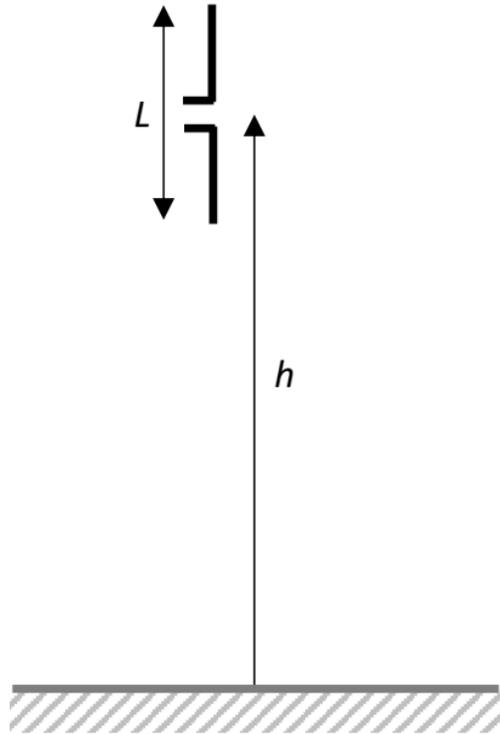


Figure 2. Single dipole above a lossy half-space

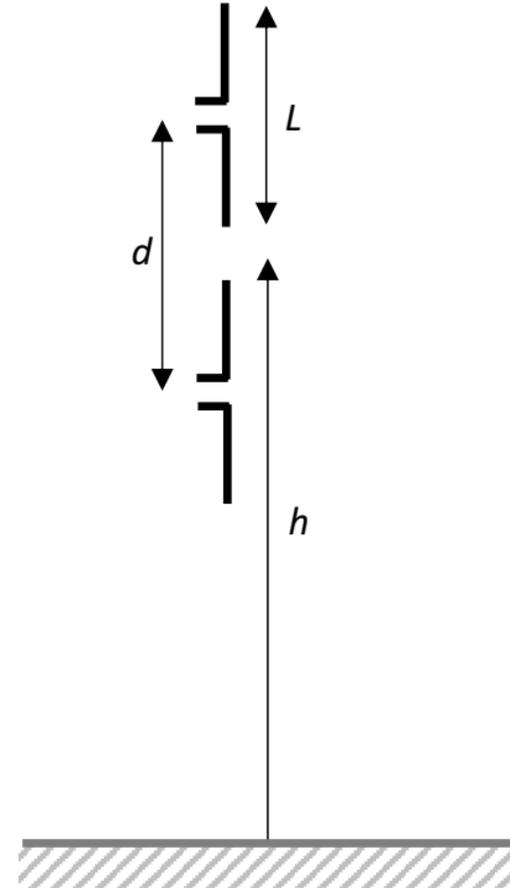


Figure 3. Two-wire array above a lossy half-space

## Mathematical model

The current distribution along the wire antenna above a lossy ground is governed by the Pocklington integro-differential equation:

$$E_z^{exc} = j\omega \frac{\mu}{4\pi} \int_{-L/2}^{L/2} I(z') g(z, z') dz' - \frac{1}{j4\pi\omega\epsilon_0} \frac{\partial}{\partial z} \int_{-L/2}^{L/2} \frac{\partial I(z')}{\partial z'} g(z, z') dz' \quad (1)$$

where the  $I(z')$  is the unknown induced current along the antenna due to the excitation field  $E_z^{exc}$  applied at the centre of the dipole.

## Mathematical model

$g(z, z')$  in equation (1) denotes the total Green function:

$$g(x, x') = g_0(x, x') + R_{TM} g_i(x, x') \quad (2)$$

where  $R_{TM}$  is Fresnel reflection coefficient for the vertical polarization:

$$R_{TM} = \frac{\underline{n} \cos \theta' - \sqrt{\underline{n}^2 - \sin^2 \theta'}}{\underline{n} \cos \theta' + \sqrt{\underline{n}^2 - \sin^2 \theta'}} \quad (3)$$

and:

$$\underline{n} = \frac{\varepsilon_{eff}}{\varepsilon_0}; \quad \varepsilon_{eff} = \varepsilon_r \varepsilon_0 - j \frac{\sigma}{\omega} \quad (4)$$

while  $g_0(z, z')$  and  $g_i(z, z')$  are corresponding Green functions:

$$g_0(z, z') = \frac{e^{-jkR_0}}{R_0}; \quad g_i(z, z') = \frac{e^{-jkR_i}}{R_i} \quad (5)$$

## Radiated electric field

The Galerkin-Bubnov scheme of the Indirect Boundary Element Method (GB-IBEM) is used for the solution of integro-differential equation (1) and corresponding current distribution is obtained.

The related radiated field is calculated using corresponding integral expressions for the irradiated fields given by:

$$E_x = \frac{1}{j4\pi\omega\varepsilon_0} \sum_{n=1}^M \int_{-L/2}^{+L/2} \frac{\partial I_n(z')}{\partial z'} \frac{\partial G_{mn}(x, z')}{\partial x} dz' \quad (6)$$

$$E_y = \frac{1}{j4\pi\omega\varepsilon_0} \sum_{n=1}^M \int_{-L/2}^{+L/2} \frac{\partial I_n(z')}{\partial z'} \frac{\partial G_{mn}(y, z')}{\partial y} dz' \quad (7)$$

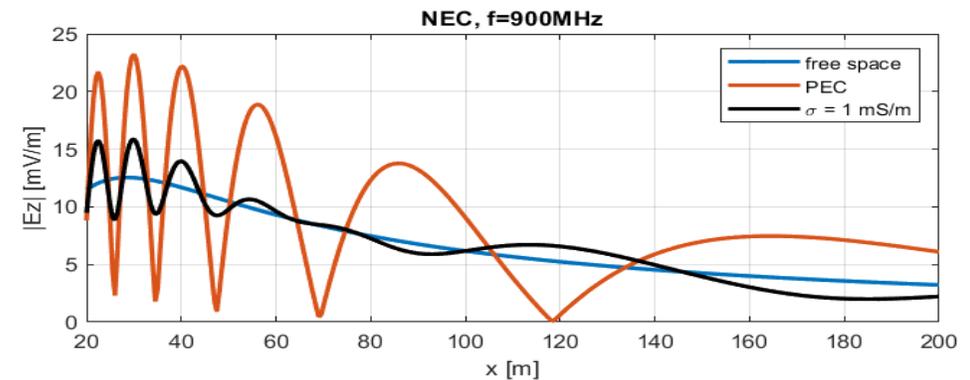
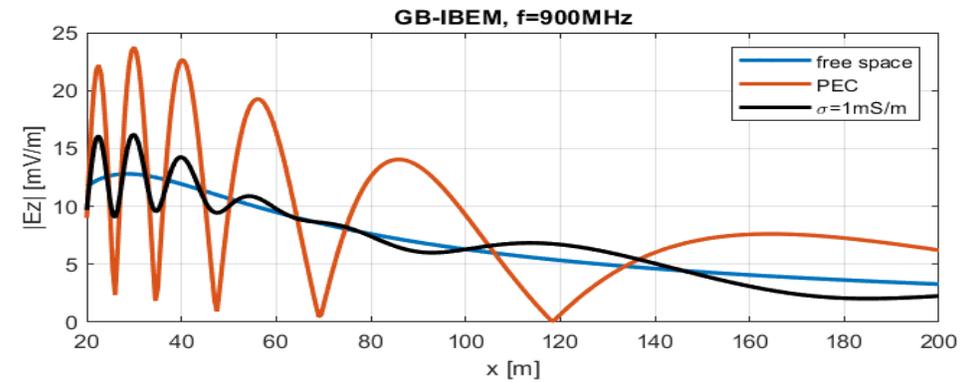
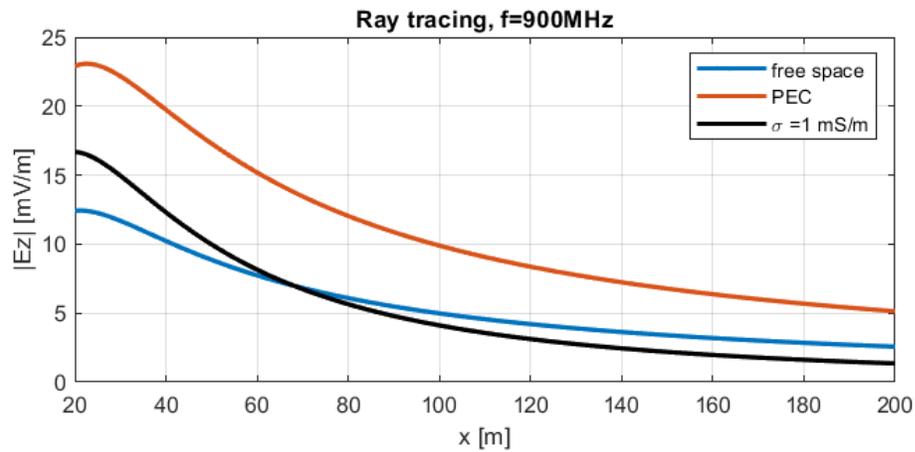
$$E_z = \frac{1}{j4\pi\omega\varepsilon_0} \sum_{n=1}^M \left[ \int_{-L/2}^{+L/2} \frac{\partial G_{mn}(z, z')}{\partial z} \frac{\partial I_n(z')}{\partial z'} dz' + k^2 \int_{-L/2}^{+L/2} I_n(z') G_{mn}(z, z') dz' \right] \quad (8)$$

## Numerical results

- Numerous calculated results are presented using different models. The obtained results are validated by comparison against the results obtained via NEC (Numerical Electromagnetic Code) package.
- First, case of a single dipole is analyzed.  $\lambda/2$  dipole is placed vertically at height  $h = 20$  m above the lossy ground ( $\sigma = 0.001$  S/m and  $\epsilon_r = 10$ ), as shown in Fig 1. The dipole is excited by 1V voltage source.

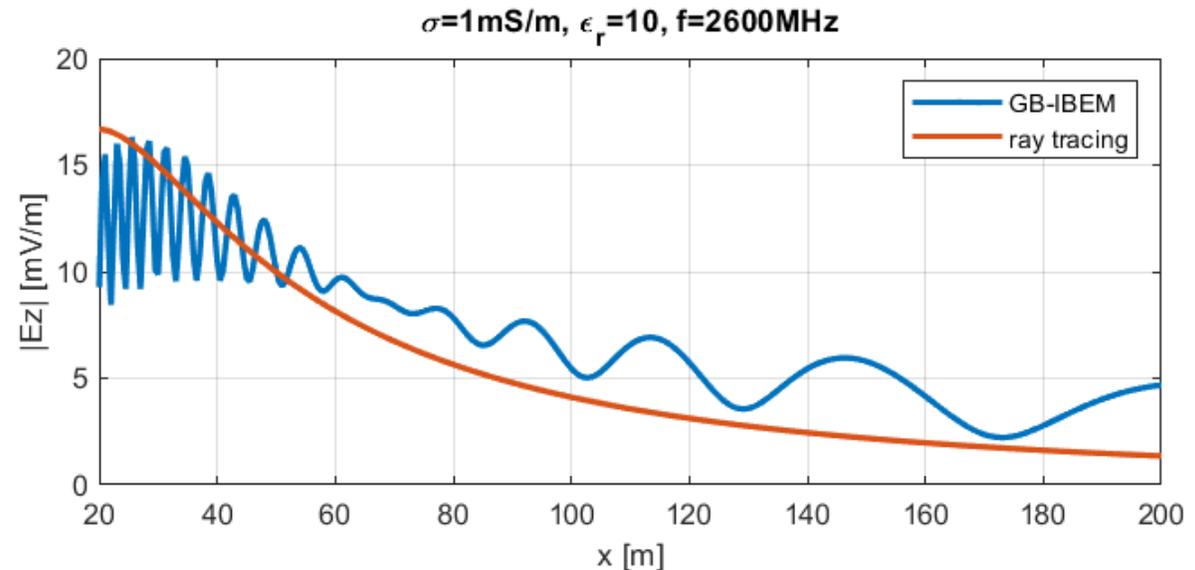
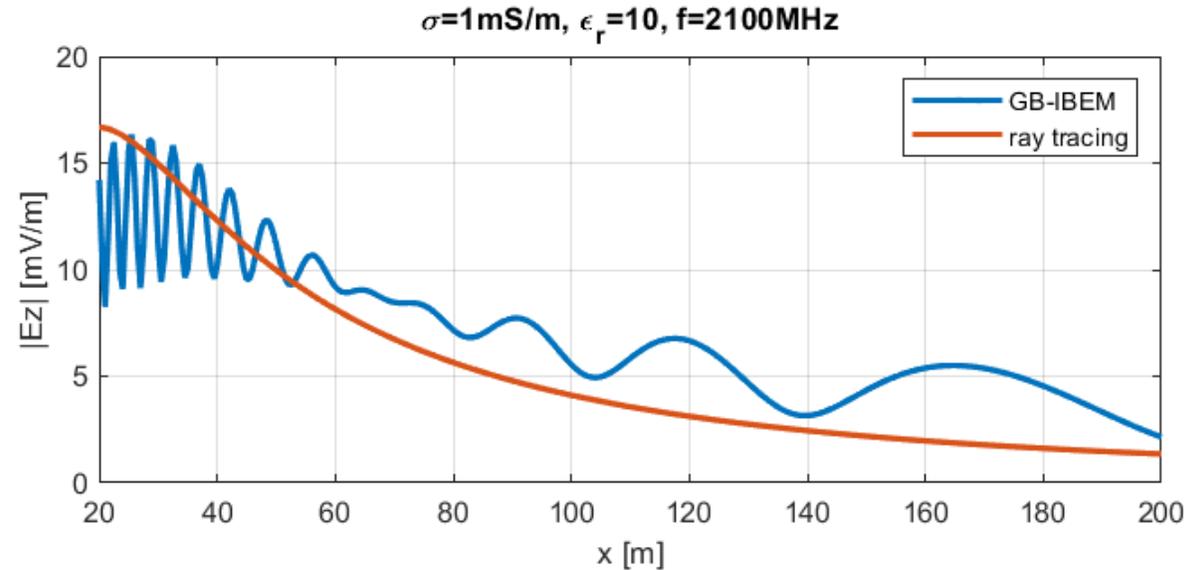
# Half wave dipole

$E_z$  field component at  $z=1.5\text{m}$  for the case of free space, PEC ground and lossy half-space ( $\epsilon_r = 10, \sigma = 1 \text{ mS/m}$ )

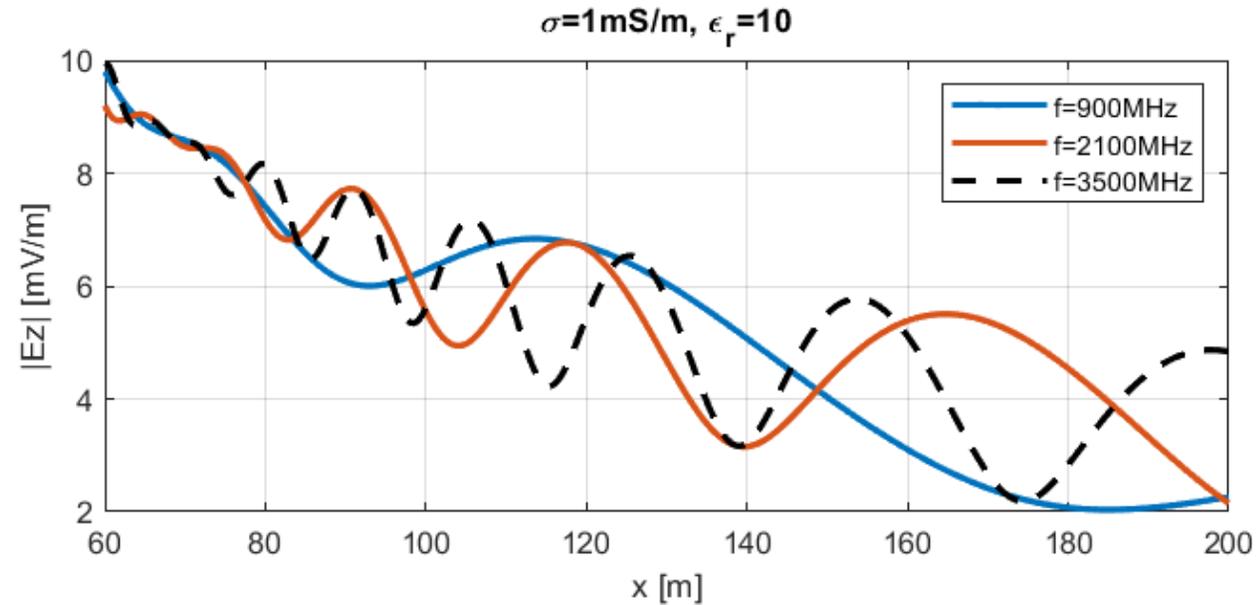


# Half wave dipole

The electric field over the real ground versus distance for LTE technology, for operating frequencies  $f=2.1\text{GHz}$  and  $f=2.6\text{GHz}$ , obtained via GB-IBEM and ray tracing.

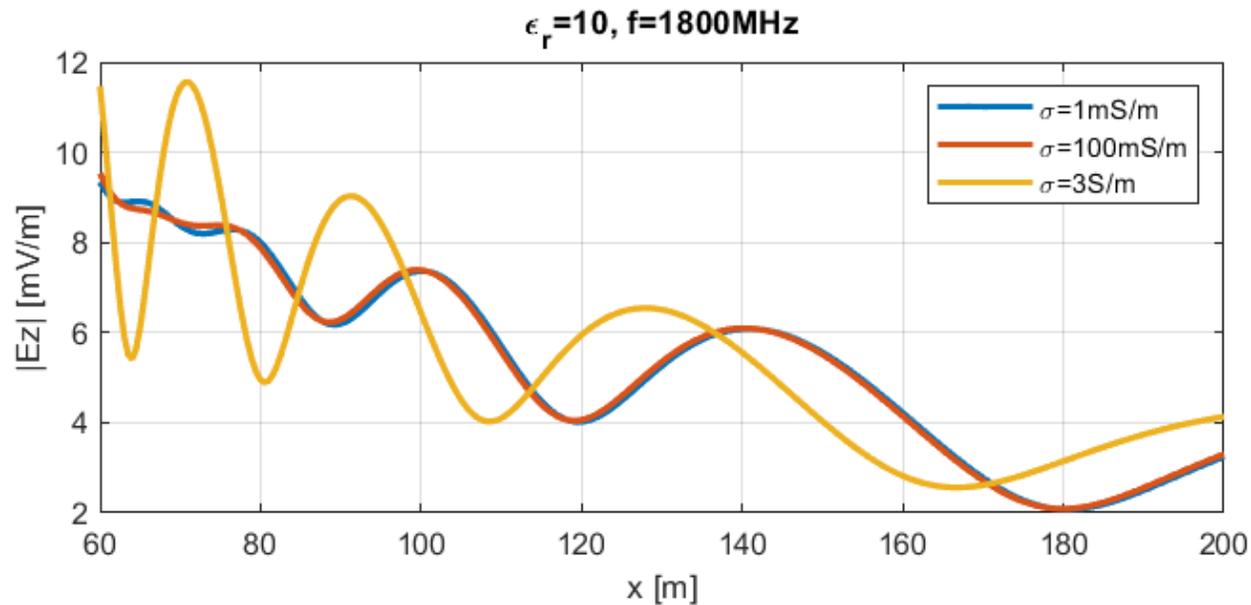


# Half wave dipole



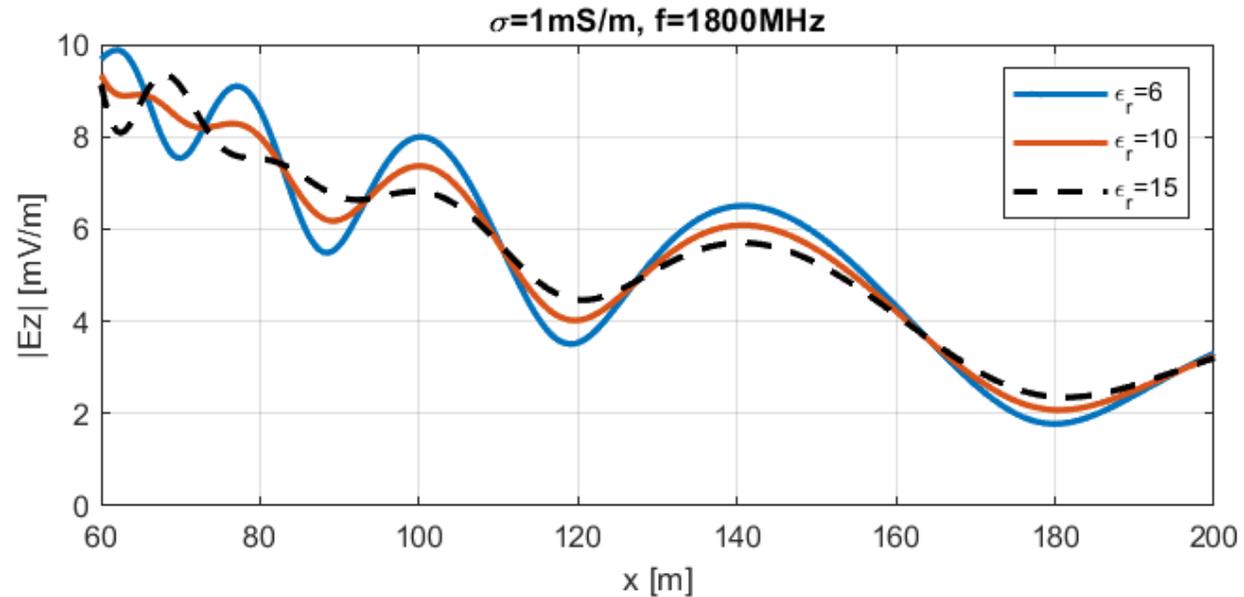
$E_z$  field component at  $z=1.5\text{m}$  for different frequencies ( $\epsilon_r = 10, \sigma = 1 \text{ mS/m}$  GB-IBEM)

# Half wave dipole



$E_z$  field component at  $z=1.5\text{m}$  for for the typical values of the ground conductivity ( $\epsilon_r = 10, \text{GB-IBEM}$ )

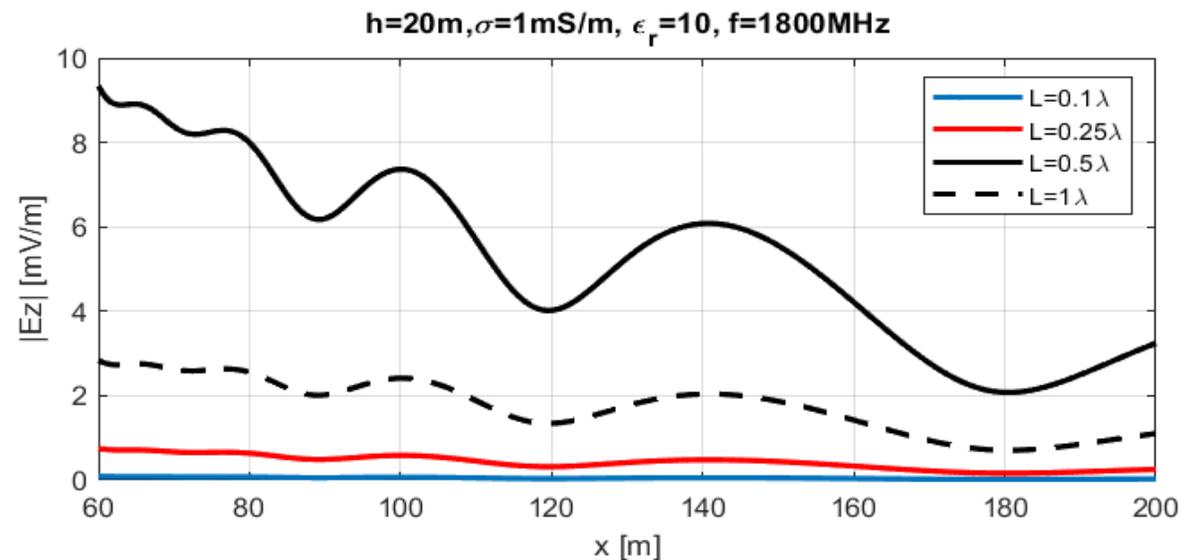
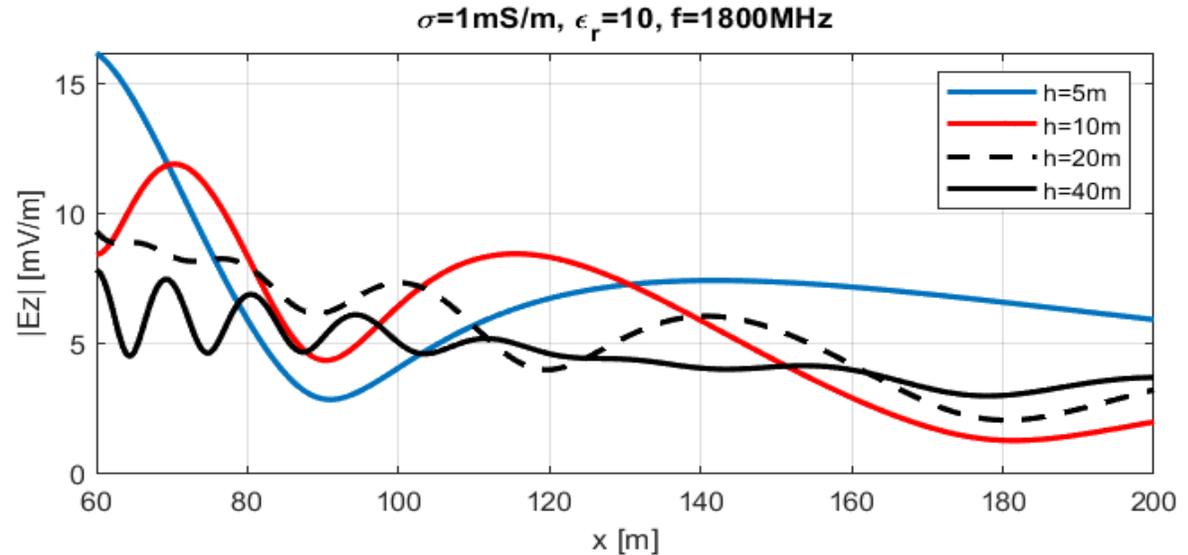
# Half wave dipole



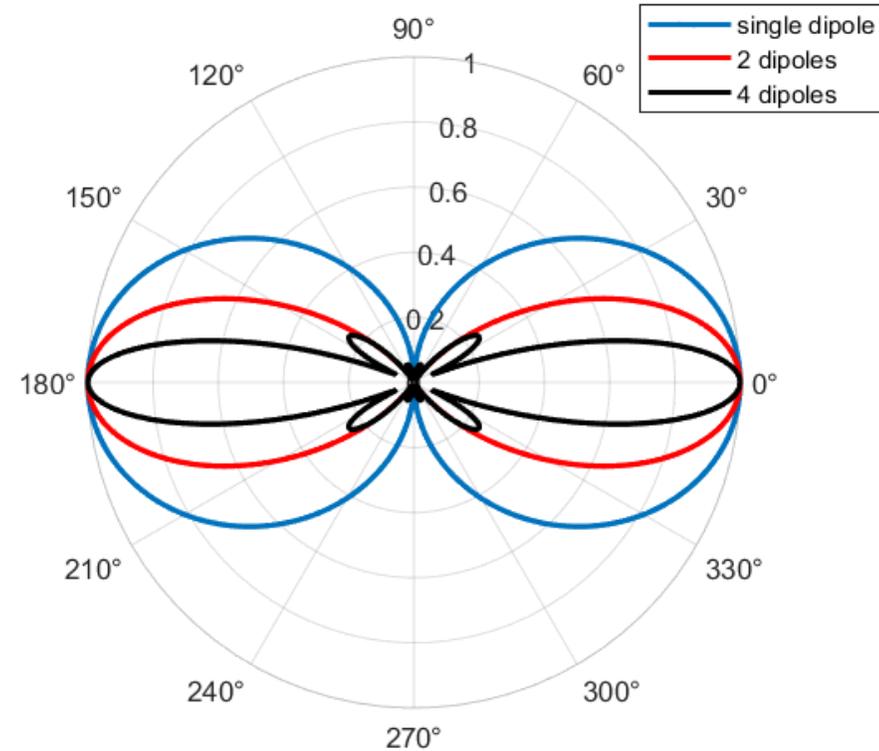
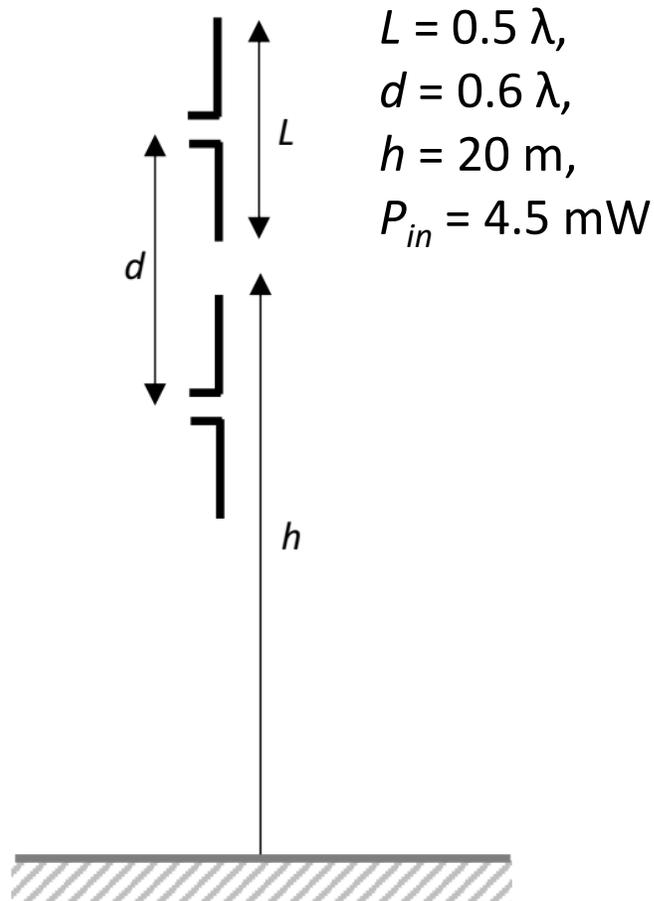
$E_z$  field component at  $z=1.5\text{m}$  for different ground permittivities ( $\sigma = 1 \text{ mS/m}$  GB-IBEM)

# Single dipole

The influence of antenna heights and lengths to the radiated field versus distance. Thus, upper Figure shows the results for different antenna heights above the interface and obtained via GB-IBEM, while lower Figure shows the results for different antenna lengths obtained by the same method.



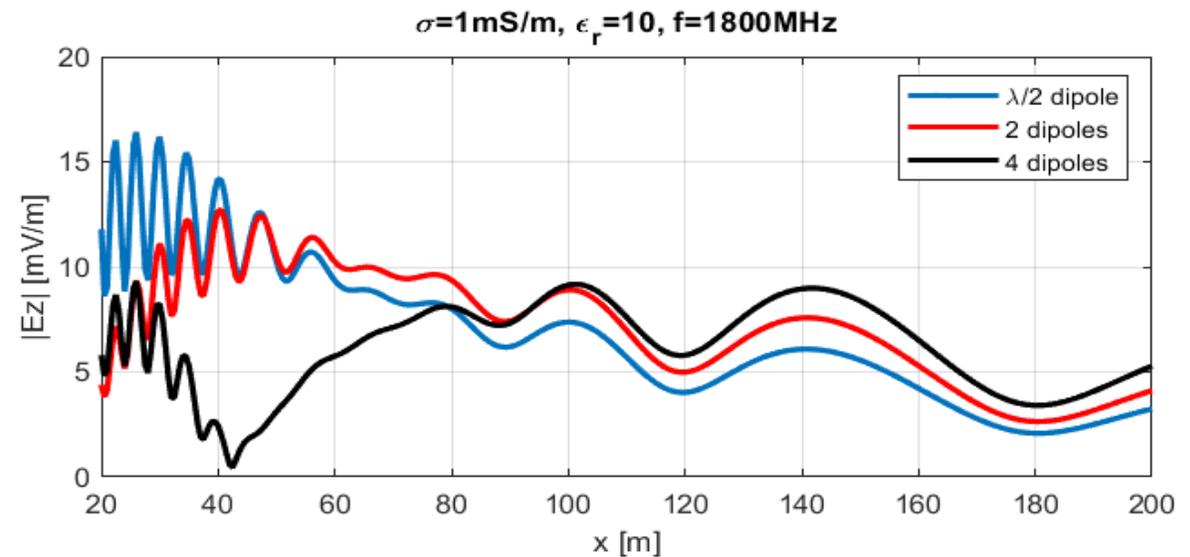
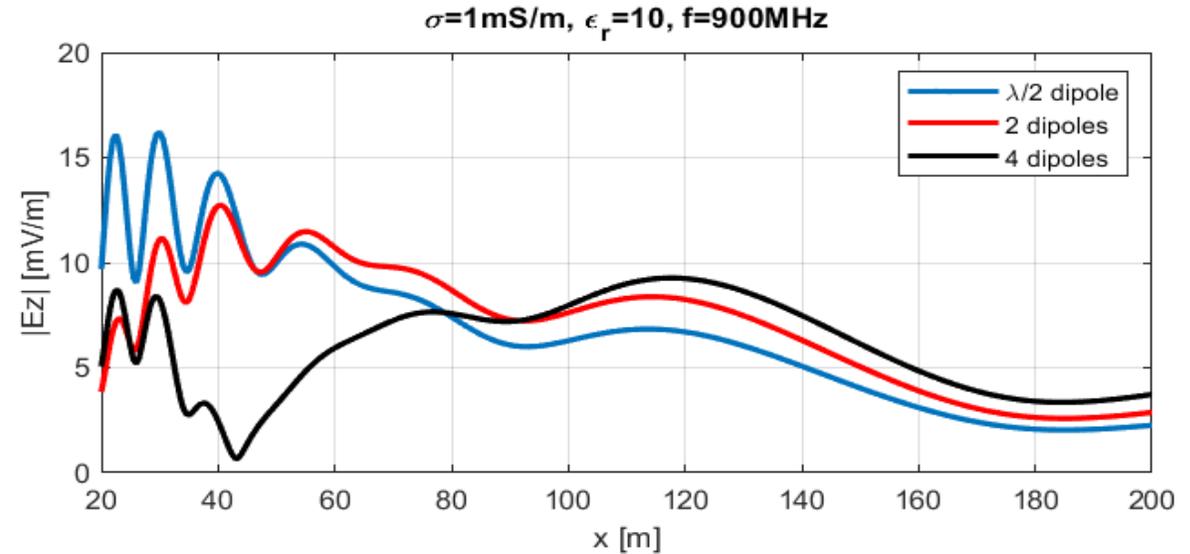
# Dipole array



Normalized radiation pattern for 3 different antenna configurations in free space

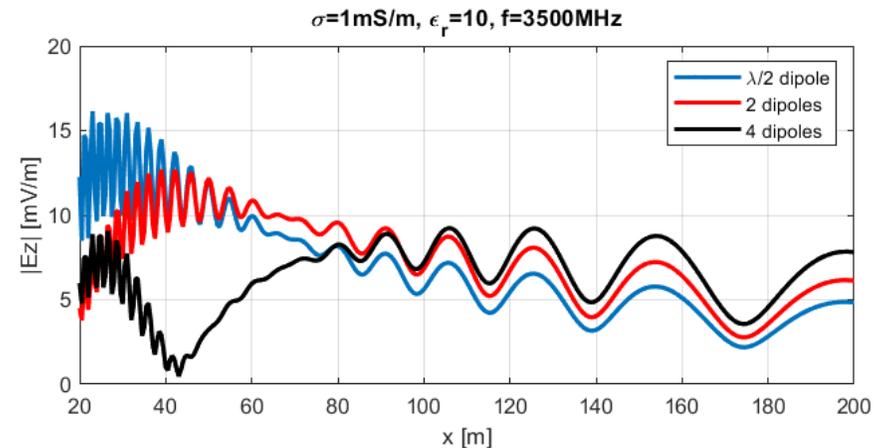
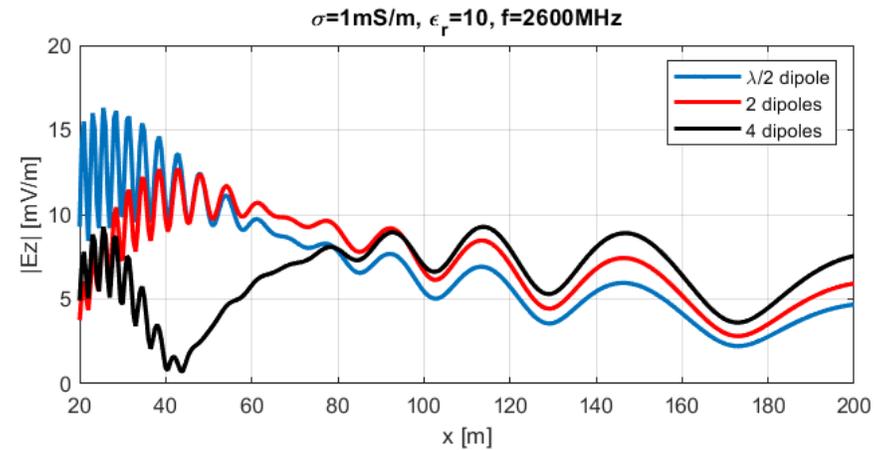
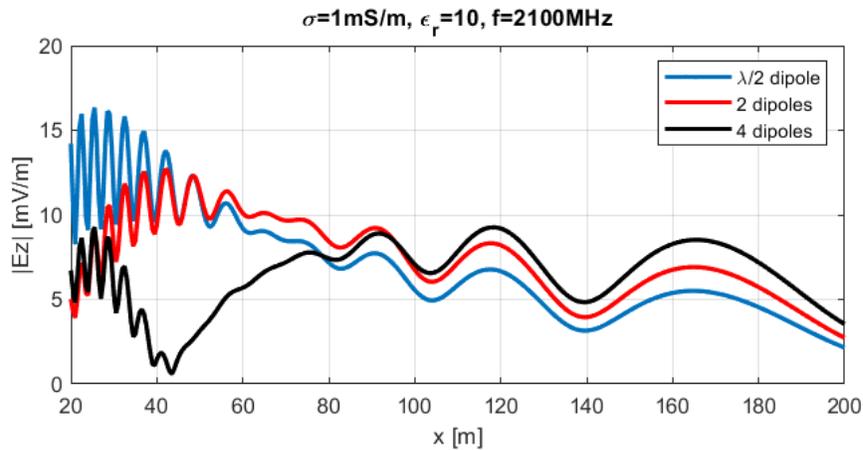
# Dipole array

$E_z$  component of the radiated field at  $z=1.5\text{m}$  for different wire array configurations, while the operating frequency is  $f=900\text{MHz}$  and  $f=1.8\text{GHz}$



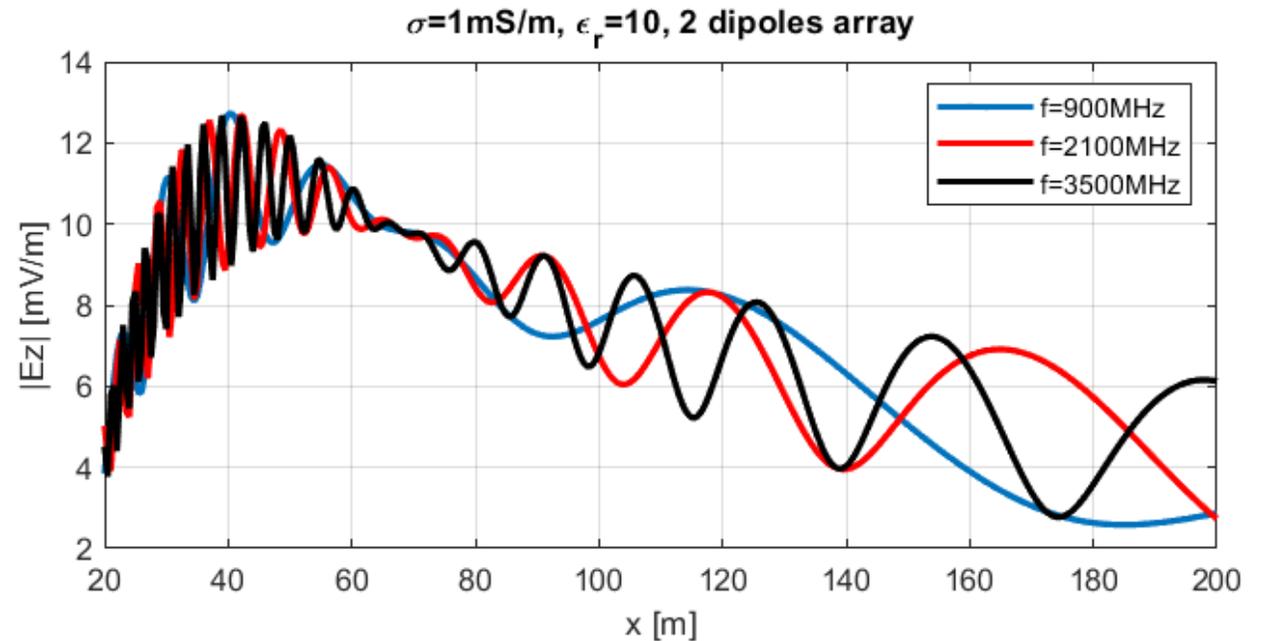
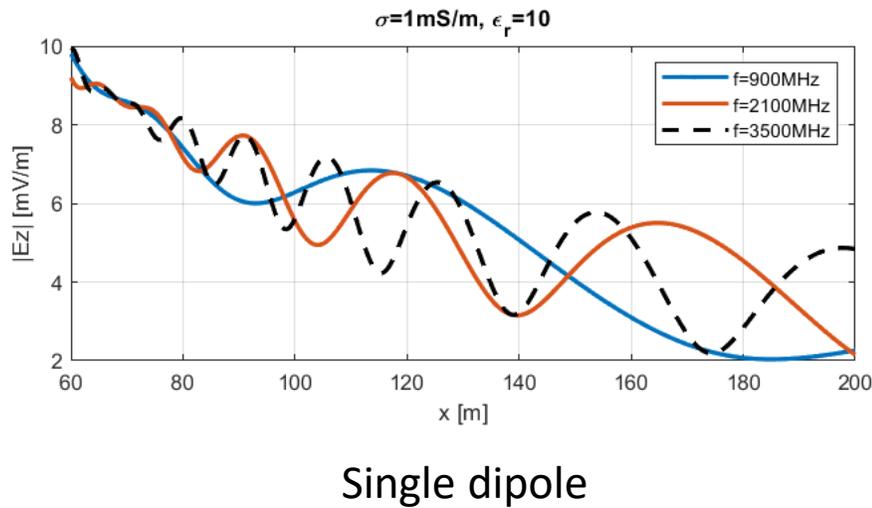
# Dipole array

$E_z$  component of the radiated field at  $z=1.5\text{m}$  for different wire array configurations, and operating frequency ( $\epsilon_r = 10, \sigma = 1 \text{ mS/m}$ )

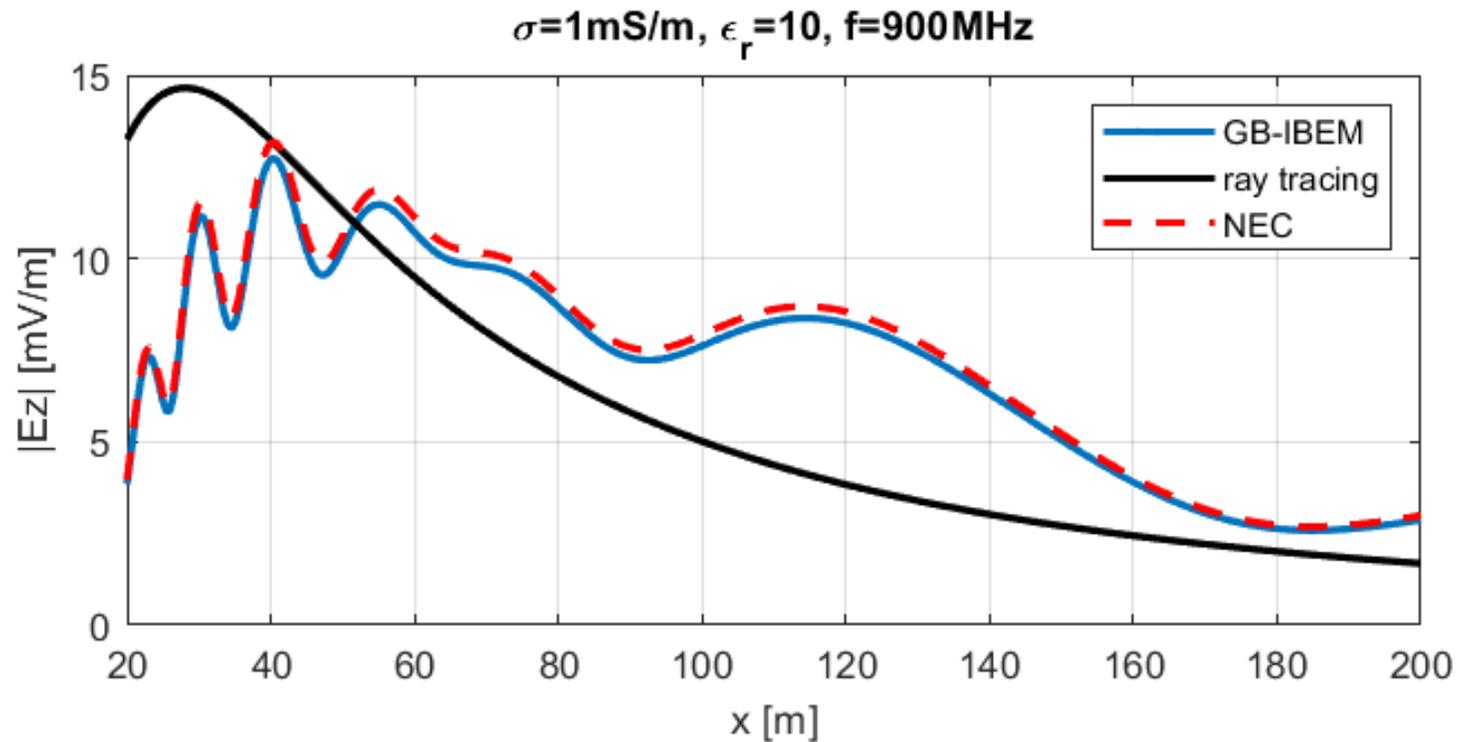


# Dipole array

$E_z$  field component at  $z=1.5\text{m}$  for different frequencies ( $\epsilon_r = 10, \sigma = 1 \text{ mS/m}$ )



## Dipole array



$E_z$  field component at  $z=1.5\text{m}$  for different methods (array of two vertical half wave dipoles)

## Concluding remarks

- The paper deals with the calculation of the electric field radiated by a base station antenna system by numerically solving Pocklington integro-differential equations arising from the wire antenna theory.
- In particular, a study on impact of the antenna length, height above ground, ground permittivity and conductivity for several operating frequency is given in the paper for the case of a single dipole, 2 and 4 dipole array, respectively.
- Results obtained via rigorous Pocklington integral equation approach and GB-IBEM solution are compared to the results obtained via NEC and simplified ray tracing technique.

## Future work

- More complex and realistic antenna systems configurations.
- Ray tracing technique will be improved by taking into account the phase shift of the propagating wave.

*Thank you*  
*for your very kind attention!*

